$$\begin{tikzcd} f \arrow[r, bend left, "\frac{d}{dx}"] & f' \arrow[l, bend left, "\text{antidifferentiation}"] \end{tikzcd}$$

<u>Ex</u>: What function(s) have $f(x) = x^3$ as its derivative?

$$\frac{d}{dx}\left[x^4\right] = 4x^3$$

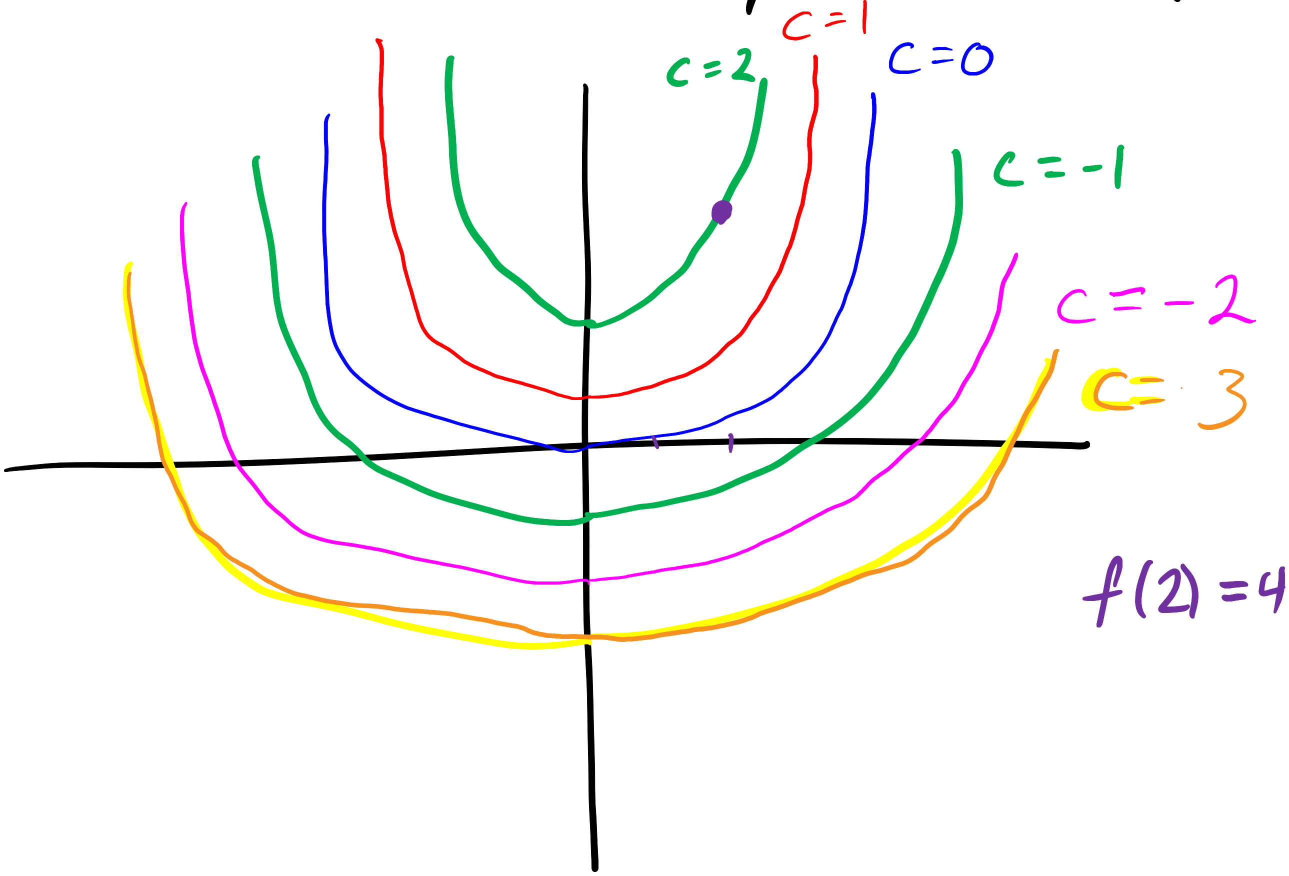
$$\frac{d}{dx}\left[\frac{1}{4}x^4\right] = x^3$$

$$\frac{d}{dx}\left[\frac{1}{4}x^4 + 6\right] = x^3$$

$$\frac{d}{dx}\left[\frac{1}{4}x^4 - 2000\right] = x^3$$

<u>Most general antiderivative</u> of $f(x) = x^3$ is $F(x) = \frac{1}{4}x^4 + C$

C is an arbitrary constant.



Theorem: If F is an antiderivative of f on I, then the most general antiderivative of f on I is

$$F(x)+C$$

where C is an arbitrary constant.

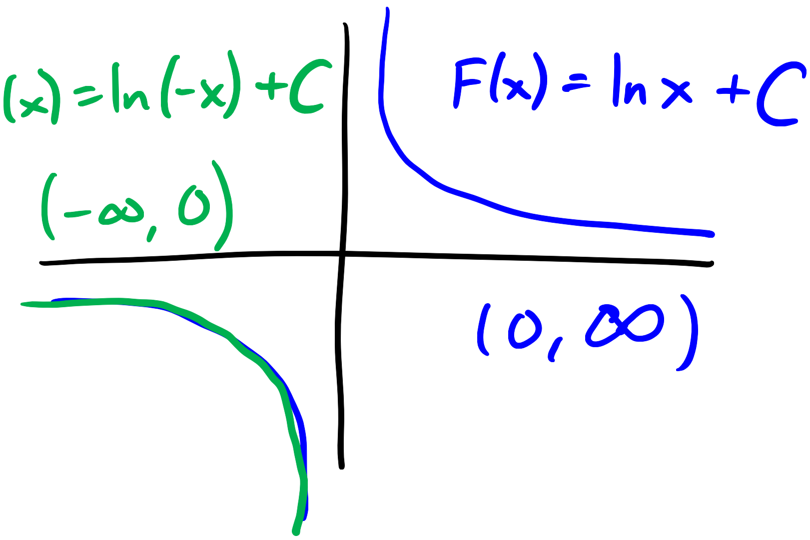
Ex: Find the most general antiderivative of $f(x)$:

① $f(x) = \cos x$

$F(x) = \sin x + C$

$F'(x) = \cos x + 0 = \cos x = f(x)$

② $f(x) = \dfrac{1}{x}$ $(x \neq 0)$



$\dfrac{d}{dx}\left[\ln(-x)\right] = \dfrac{1}{-x} \cdot (-1) = \dfrac{1}{x}$

$(-\infty, 0)$

$\dfrac{d}{dx}\left[\ln|x|\right] = \dfrac{1}{x}$

$F(x) = \ln|x| + C$

$(x \neq 0)$

③ $f(x) = x^n$ $(n \neq -1)$

$\dfrac{d}{dx}\left[x^{n+1}\right] = (n+1)\, x^n \longrightarrow \dfrac{d}{dx}\left[\dfrac{x^{n+1}}{n+1}\right] = x^n$

So $F(x) = \dfrac{x^{n+1}}{n+1} + C$

function	most general antiderivative
$f(x)$	$F(x)$
$cf(x)$	$cF(x)$
$f(x) \pm g(x)$	$F(x) \pm G(x)$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + C$
e^x	$e^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

$\sec x \tan x$	$\sec x + C$
$\sec^2 x$	$\tan x + C$

Ex: Find the general antiderivative:

① $f(x) = x^2 - 3x + 2$

$F(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C$

② $f(x) = \sqrt{2}$

$F(x) = x\sqrt{2} + C$

③ $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$

$= 3x^{1/2} - 2x^{1/3}$

④ $f(x) = \frac{1}{5} - \frac{2}{x}$

$F(x) = \frac{1}{5}x - 2\ln|x| + C$

⑤ $h(\theta) = 2\sin\theta - \sec^2\theta$

$H(\theta) = -2\cos\theta - \tan\theta + C$

$F(x) = 2\cdot(\sqrt{x})^3 - \frac{3}{2}(\sqrt[3]{x})^4 + C$

$= 2x^{3/2} - \frac{3}{2}x^{4/3} + C$

Ex : Find the particular antiderivative of

$f'(x) = 5x^{2/3}$ satisfying $f(8) = 21$. initial value

$$f(x) = 5x^{5/3} \cdot \frac{1}{5/3} + C$$

$$8^{5/3} = (8^{1/3})^5$$

$$= 3x^{5/3} + C$$

$$f(8) = 3(8)^{5/3} + C = 3(2)^5 + C = \boxed{96 + C = 21}$$

$$C = -75$$

Particular antiderivative : $\boxed{f(x) = 3x^{5/3} - 75}$

$a(t) = -g$

$v(0) = v_0$

$y(0) = y_0$

$$y = -\frac{1}{2} g t^2 + v_0 t + y_0$$

$y' = v$, $y'' = a$